Exponential Functions in Context (2.4+2.5) Homework

1. Determine the end behavior of the following function: $f(x) = 3(e)^{-x+2}$. Select one.
   (A) As the input values of $f(x)$ decrease without bound, the output values get arbitrarily close to 0. As the input values of $f(x)$ increase without bound, the output values increase without bound.

   (B) As the input values of $f(x)$ decrease without bound, the output values get arbitrarily close to 0. As the input values of $f(x)$ increase without bound, the output values decrease without bound.

   (C) As the input values of $f(x)$ decrease without bound, the output values increase without bound. As the input values of $f(x)$ increase without bound, the output values get arbitrarily close to 0.

   (D) As the input values of $f(x)$ decrease without bound, the output values decrease without bound. As the input values of $f(x)$ increase without bound, the output values get arbitrarily close to 0.

2. Let $g(x) = -5\left(\frac{1}{e}\right)^x$
   a. Is $g(x)$ always positive or always negative?
   b. Is $g(x)$ always increasing or always decreasing?
   c. Is $g(x)$ always concave up or always concave down?
   d. Is the rate of change of $g(x)$ always positive or always negative?
   e. Is the rate of change of $g(x)$ always increasing or always decreasing?

3. Before the industrial revolution, the earth's atmosphere contained carbon dioxide at a level of 280 parts per million. Since then, the amount of carbon dioxide in the earth's atmosphere has been rising by approximately 0.17% every year.
   a. Determine a function that models the amount of carbon dioxide in the atmosphere t years after the industrial revolution.
   b. The last year that the earth's atmosphere contained carbon dioxide at a level of 280 parts per million was 1760. Use a calculator to predict the level of carbon dioxide in the earth's atmosphere in the year 2025.

4. Transistors are a key component of digital electronics. As technology has progressed, transistors have gotten smaller. Fifty years ago, Gordon Moore (founder of Intel) predicted that the number of transistors that can fit on a microchip would double every two years. At the time of his prediction, 5000 transistors fit on a microchip. Assuming he was correct, let $C(t)$ model how many transistors fit on a microchip t years after Moore's prediction. Which one of the following could represent $C(t)$?
   (A) $C(t) = 5000\left(\sqrt{2}\right)^{2t}$
   (B) $C(t) = 5000(2)^{-2t}$
   (C) $C(t) = 5000(2)^{2t}$
   (D) $C(t) = 5000(2)^{\frac{t}{2}}$

5. Warfarin is a blood thinner used to prevent blood clots, reducing the risk of stroke or heart attack. Every day, about 10% of the warfarin in a body is metabolized. That is to say, if 5 mg of warfarin is in one's body Monday morning, only 4.5 mg remains Tuesday morning.
   a. Mark starts with D mg of warfarin in his body. Write a function that represents how much is left t days later.
      (A) $f(t) = D(0.1)^t$
      (B) $f(t) = D(0.9)^t$
      (C) $f(t) = D(1.1)^t$
      (D) $f(t) = D(10)^t$

   b. Mark starts with D mg of warfarin in his body. Write a function that represents how much is left t weeks later.

6. Lead-217 is a lead isotope ($^{217}_{82}Pb$ for you chemistry nerds) with a half-life of 20 seconds. That means that every 20 seconds, the amount of lead-217 in a sample is halved as the radioactive isotope breaks down into daughter isotopes.
   a. If a sample initially has 100 picograms of lead-217, which one of the following models the amount of lead-217 left in the sample, in picograms, after t **seconds**?
      (A) $2000\left(\frac{1}{2}\right)^{t}$
      (B) $5\left(\frac{1}{2}\right)^{t}$
      (C) $100\left(\frac{1}{2}\right)^{\frac{t}{20}}$
      (D) $100\left(\frac{1}{2}\right)^{20t}$
   b. If a sample initially has 100 picograms of lead-217, which one of the following models the amount of lead-217 left in the sample, in picograms, after t **minutes**?
      (A) $300\left(\frac{1}{2}\right)^{t}$
      (B) $100\left(\frac{1}{2}\right)^{\frac{t}{3}}$
      (C) $100\left(\frac{1}{6}\right)^{t}$
      (D) $100\left(\frac{1}{2}\right)^{3t}$

7. In 2000, the price of the prince's diamond was $\$25{,}000$. Each year the price increases by 5%. Write a function modelling the price of the diamond t years after the year 2000.

8. Alberto the alligator has an algae problem. There are 50 lbs of algae in his home. He invites some grass shrimp to live with him as they are known for their ability to eat algae. Each week, the level of algae decreases by 3%.
   a. Write a function modelling the weight of algae in Alberto's home t weeks after inviting the algae shrimp to live with him.
   b. Write a function modelling the weight of algae in Alberto's home t days after inviting the algae shrimp to live with him.

9. Alberto the alligator has a grass shrimp problem. Inviting them in was a mistake. His home is now infested with them. Today there are 400 grass shrimp in Alberto's home, but every 30 days, the population doubles. Write a function modelling the number of grass shrimp in Alberto's home t days from today.

Let $f(x) = 3^x$. Match each of the following numbered equations with the letter of the corresponding description. You may have to use exponent properties to rewrite the numbered equations in an equivalent form before you realize which lettered description is correct. Or start with the description, write a function, then match.

10. $g(x) = 9(3)^x$
11. $h(x) = 9^x$
12. $k(x) = 3^{x-1}$
13. $m(x) = \left(\frac{1}{3}\right)^x$

(A) The graph of this function is the image of the graph of $f(x)$ after a horizontal dilation by a factor of $\frac{1}{2}$.
(B) The graph of this function is the image of the graph of $f(x)$ after a reflection over the y-axis.
(C) The graph of this function is the image of the graph of $f(x)$ after a horizontal translation by -2 units.
(D) The graph of this function is the image of the graph of $f(x)$ after a vertical dilation by a factor of $\frac{1}{3}$.

Rewrite in the form $a(b)^x$

14. $f(x) = 24(3)^{2x-1}$

15. $g(x) = (10)(2)^{2x}(9)^{\frac{1}{2}x}$

Exponential Functions in Context (2.4+2.5) HW Answers

1. C
2.
    a. Negative
    b. Increasing
    c. Concave Down
    d. Positive (rate of change is positive since $f(x)$ is increasing)
    e. Decreasing (rate of change is decreasing since $f(x)$ is concave down)
3.
    a. $C(t) = 280(1.0017)^t$
    b. If 1760 corresponds with $t = 0$ and t is measured in years, then $t = 265$ represents 2025 (2025 is 265 years after 1760).
    $$C(265) = 280(1.0017)^{265} = 439.179$$
    439.179 parts per million is the carbon dioxide level in 2024 according to the model.
4. D
5.
    a. B
    b. $f(t) = D(0.9)^{7t}$
6.
    a. C
    b. D
7. $P(t) = 25000(1.05)^t$
8.
    a. $W(t) = 50(0.97)^t$
    b. $W(t) = 50(0.97)^{\frac{t}{7}}$
9. $P(t) = 400(2)^{\frac{t}{30}}$
10. C
11. A
12. D
13. B
14. $f(x) = 8(9)^x$
15. $g(x) = 10(12)^x$